تم الرفع بواسطة د. معن أبو عيسى

Calculus 2
~~Second~~
First

79/100

RTN 12:00 - 1:00

Palestine Technical University
Department of Applied Mathematics.
Calculus II
First Exam

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Student name بالعربيه: Apr, 14^{th}, 2010

Student #: Duration: 70 min.

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Q1. (30 points) Choose the correct answer.

$g = f^{-1}(x) \quad g(-2) = f^{-1}(2) = 5$

$(f^{-1}(2))' = \frac{1}{f'(5)}$

(1) Let f and g be two differentiable functions. If g is the inverse of f and if $g(-2) = 5$ and $f'(5) = \frac{-1}{2}$, then $g'(-2) =$

$(f^{-1}(-2))' = \frac{1}{f'(5)} = \frac{1}{f'(f^{-1}(b))} \quad f(5) = -2 \quad f^{-1}(5) = -2$

(a)2 (b) -2

(c) $\frac{-1}{2}$ (d) None of the above

(2) The value of $\lim_{x \to \infty}\left(x \tan^{-1}\frac{2}{x}\right) =$

$\frac{\tan^{-1}(\frac{2}{x})}{\frac{1}{x}} \quad \frac{\frac{1}{1+\frac{4}{x^2}}}{\frac{-1}{x^2}} = \frac{\frac{1}{1+0}}{\frac{-1}{\infty}} = \frac{1}{0} = \infty$

(a) 1/2 (b) 1

(c) 2 (d) ∞

(3) The value of $2^{\log_4 9} =$

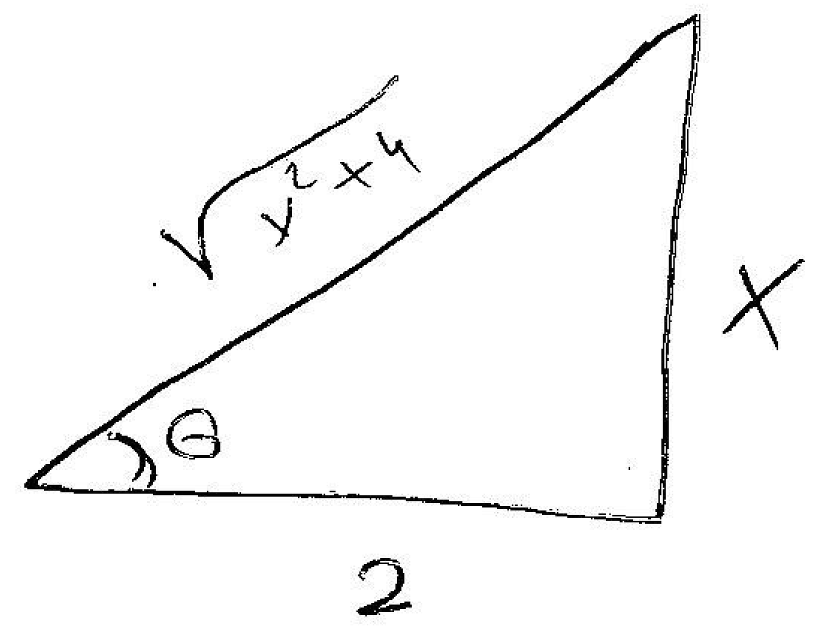
$4^{\frac{1}{2}\log_4 9} \quad 4^{\log_4 9^{\frac{1}{2}}} \quad 4^{\log_4 3}$

(a) 3 (b) 9

(c) 9/4 (d) $\frac{\ln 9}{\ln 4}$

(4) The value of $\sec\left(\tan^{-1}\frac{x}{2}\right) =$

$\frac{1}{\cos\theta}$



(a) $\frac{x}{\sqrt{x^2+4}}$

(b) $\frac{2}{x}$

(c) $\frac{\sqrt{x^2+4}}{2}$

(d) $\frac{2}{\sqrt{x^2+4}}$

$\frac{1}{\frac{2}{\sqrt{x^2+4}}}$

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(a)

$\sqrt{\ln x}\Big]_2^{16}$ $\quad \sqrt{\ln 16}-\sqrt{\ln 2}$

$(\ln 16)^{\frac12}-(\ln 2)^{\frac12}$

$\frac12 \ln 2^4$ $\quad \frac12\ln 16-\frac12 \ln 2$

$2\ln 2 - \sqrt{\ln 2}$

(5) If $y=\tan^{-1}(\ln x)$, then $\dfrac{dy}{dx}$ at $x=e$ is

(a) $\dfrac{\pi}{4}$ (b) $\dfrac{1}{1+e^2}$

(c) $\dfrac{1}{4e}$ (d) $\dfrac{1}{2e}$

$y=\tan^{-1}(\ln x)$

$y'=\dfrac{1}{1+(\ln x)^2}\cdot\dfrac{1}{x}$

$\dfrac{1}{1+1}\cdot\dfrac{1}{e}=\dfrac12\cdot\dfrac1e=\dfrac{1}{2e}$

(6) The value of the integral $\displaystyle\int_0^2 \frac{dt}{8+2t^2}=$

(a) $\dfrac{\pi}{16}$ (b) $\dfrac{\pi}{4}$

(c) $\dfrac{\pi}{12}$ (d) π

$\dfrac{\pi}{4}$

$\sqrt8 \quad \sqrt2 t$

$a^2+u^2 \quad = \tan^{-1}\dfrac{\sqrt2 t}{\sqrt8}$

$u=2t$

$du=2dt$

$\tan^{-1}\dfrac{\sqrt2 t}{\sqrt8}\Big]_0^2$ $\quad \sqrt{2t^2}=\sqrt2 t$ $\quad \dfrac12\displaystyle\int\frac{du}{8+u^2}$

$\tan^{-1}(1)-\tan^{-1}(0)$ $\quad \dfrac12\tan^{-1}\left(\dfrac{u}{\sqrt8}\right)$

$\sqrt8 \quad \sqrt2 \quad 1$

(7) The value of the integral $\displaystyle\int_2^{16}\frac{dx}{2x\sqrt{\ln x}}=$

(a) $2-\sqrt{\ln 2}$ (b) $\sqrt{\ln 2}$

(c) $\ln 2-\ln 5$ (d) $3\ln 2$.

$=\dfrac12\tan^{-1}\left(\dfrac{u}{\sqrt8}\right)=\dfrac12\tan^{-1}\left(\dfrac{2t}{\sqrt8}\right)$

$\dfrac12\left[\tan^{-1}\left(\dfrac{4}{\sqrt8}\right)-\tan^{-1}(0)\right.$

$\dfrac12\left[\tan^{-1}(\sqrt2)-0\right.$

$\sqrt u \quad u=\ln x \quad du=\frac1x dx$

$\frac12\int\frac{1}{\sqrt u}\quad \displaystyle\int\frac{du\, x}{2x\sqrt u}$

(8) Let $f(x)=\dfrac{e^x+e^{-x}}{2}$. The value of the integral $\displaystyle\int_1^2\frac{f(\ln(x))}{x}dx=$

(a) 3/4 (b) 1/2

(c) 1/4 (d) 1

$\dfrac{u^{-\frac12+1}}{-\frac12+1}$

$\sqrt u\,\Big]$ $\quad \sqrt{\ln x}\Big]_2^{16}$

$\dfrac{e^{\ln x}+e^{-\ln x}}{2}=\dfrac{\frac{x}{1}+\frac1x}{2}=\dfrac{\frac{x^2+1}{x}}{2}$

$\dfrac{x^2+1}{x}\times\dfrac12$

(9) If $f(x)=\ln(\sec^2 x)$, then $f'(x)=$

(a) $\ln(\tan x)$

(b) $2\sec x$

(c) $\ln(2\sec x)$

(d) $2\tan x$

$\dfrac{1}{\sec^2 x}\cdot(2\sec x)\cdot(\sec x\tan x)$

$\sqrt{\ln 16}$

$\dfrac{\frac{x^2+1}{2x}}{x}$ $\quad \dfrac{x^2+1}{2x^2}=\dfrac12+\dfrac{1}{2x^2}$

$(-1,1)$

$-\frac{\pi}{2},\frac{\pi}{2}$

10. $\sin^{-1}\sin\dfrac{3\pi}{4}=$

(a) $-\dfrac{3\pi}{4}$

(b) $\dfrac{3\pi}{4}$

(c) $\dfrac{\pi}{4}$

(d) $-\dfrac{\pi}{4}$

135

$\sin 135°$

$180-135$

$\sin 45$

$\dfrac{1}{\sqrt2}$

$\sin^{-1}\left(\dfrac{1}{\sqrt2}\right)$

$\ln x=u$

$du=\frac1x dx \quad (\ln 16)^{\frac12}$

$\displaystyle\int\frac{du\,x}{2x\sqrt u}\quad -(\ln 2)^{\frac12}$

$\frac12\ln 2^4$

$2\ln 2$

$-\dfrac12\displaystyle\int\frac{du}{\sqrt u}$ $\quad 2\ln 2$

$2\ln 2-\sqrt{\ln 2}$

$\sqrt{\ln 2}(2\sqrt{\ln 2}-1)$

$\frac12\displaystyle\int\frac{u^{-\frac12+1}}{-\frac12+1}$ $\quad \frac{1}{2}\cdot 2u^{\frac12}$

$\frac12\dfrac{u^{\frac12}}{\frac12}$ $\quad u^{\frac12}$

$\displaystyle\int\frac12+\frac12 x^{-2}$

$\frac12 x+\frac12\dfrac{x^{-2+1}}{-2+1}$

$\frac12 x-\frac12\dfrac{1}{x}$

$\frac12 x-\frac{1}{2x}\Big]_1^2$

$\left[1-\frac14\right]-\left[\frac12-\frac12\right]$

$(\ln 16)^{\frac12}-(\ln 2)^{\frac12}$

$\frac12\ln 2^4-\frac12\ln 2$

$\sqrt{\ln 16}-\sqrt{\ln 2}$ $\quad \sqrt{\ln x}\Big]_2^{16}=\sqrt{\ln 16}-\sqrt{\ln 2}$

$\dfrac{1}{\sqrt2}$ $\quad \sqrt2 \quad 45° \quad 1$

$(\cancel{4x-x^2})(4x-x^2)$

$\cancel{16x^2 - 4x^3} - 4x^3 + x^4$

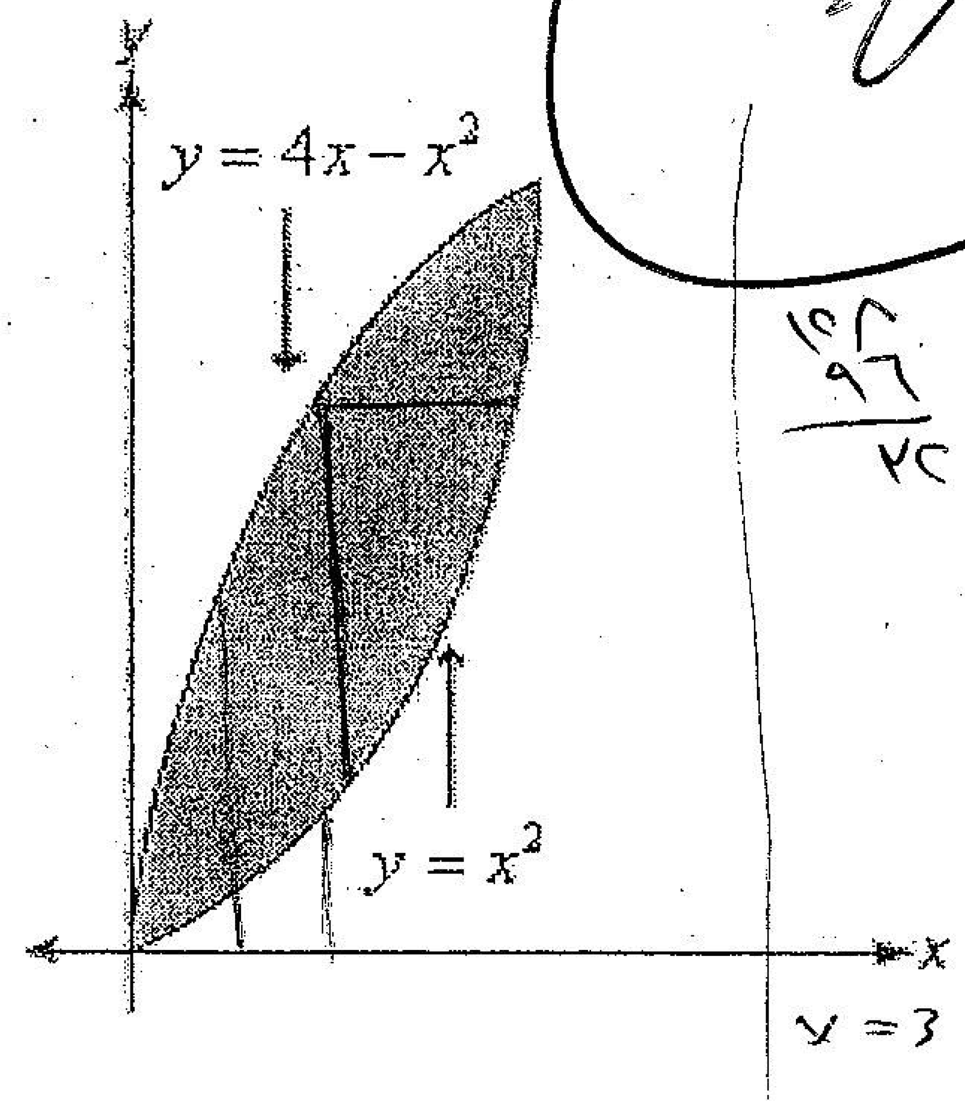
$\cancel{x^4 - 8x^3 + 16x^2}$

معتصم سلام قحطان

$\frac{22}{23}$

Q2. (23 points) Let $\boldsymbol{R}$ be the region bounded by $y = x^2$ and $y = 4x - x^2$.
Find the volume of the solid of revolution generated when $\boldsymbol{R}$ is revolved around

(a) The x-axis. (13 pts.)



we'll use washer Methed

inner $= \boxed{x^2}$

outer $= \boxed{4x - x^2}$

$r^2(x) = (x^2)^2 = \boxed{x^4}$

$R^2(x) = (4x - x^2)^2 = \boxed{x^4 - 8x^3 + 16x^2}$

Point of intersection : $4x - x^2 = x^2$

$4x - 2x^2 = 0 \Rightarrow 2x(2-x) = 0$

$\boxed{x = 0 \;,\; x = 2}$

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$$V = \pi \int_0^2 (x^4 - 8x^3 + 16x^2) - x^4 = \pi \int_0^2 -8x^3 + 16x^2 = \pi \left[-\frac{8x^4}{4} + \frac{16x^3}{3} \right]_0^2$$

$$= \pi \left[-2x^4 + \frac{16x^3}{3} \right]_0^2 = \pi \left[\left(-2(16) + \frac{16(8)}{3} \right) - (0) \right]$$

$$= \pi \left[-32 + \frac{128}{3} \right] = \pi \left[-\frac{96}{3} + \frac{128}{3} \right] = \pi \left[\frac{32}{3} \right] = \boxed{\frac{32}{3}\pi} \text{ units}$$

(b) The line $x = 3$. (10 pts.)

we'll use Shell Methed

radius $= \boxed{(3-x)}$

height $= 4x - x^2 - x^2 = \boxed{4x - 2x^2}$

Thickness $= dx$

9

$$V = 2\pi \int_0^2 (3-x)(4x - 2x^2)\, dx = 2\pi \int_0^2 12x - 6x^2 - 4x^2 + 2x^3$$

$$= 2\pi \int_0^2 12x - 10x^2 + 2x^3 = 2\pi \left[\frac{12x^2}{2} - \frac{10x^3}{3} + \frac{2x^4}{4} \right]_0^2$$

$$= 2\pi \left[6x^2 - \frac{10}{3}x^3 + \frac{x^4}{2} \right]_0^2$$

$$= 2\pi \left[24 - \frac{10}{3}(8) + \frac{16}{2} \right] - (0)$$

$$= 2\pi \left[24 + 8 - \frac{80}{3} \right] = 2\pi \left[32 - \frac{80}{3} \right]$$

$$= 2\pi \left[\frac{96}{3} - \frac{80}{3} \right] = 2\pi \left[\frac{16}{3} \right] = \boxed{\frac{32}{3}\pi \text{ units}}$$

معمر سلام مقدان

Q3. (23 points)

(a) Use logarithmic differentiation to find the derivative of (15 pts.)

$$y=\sqrt[3]{\frac{x(x+1)^9(x-2)}{(x^2+1)(2x+3)^6}}.$$

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$$\ln y = \ln\left(\frac{x(x+1)^9(x-2)}{(x^2+1)(2x+3)^6}\right)^{\frac{1}{3}}$$

$$\ln y = \frac{1}{3}\left[\ln x + \ln(x+1)^9 + \ln(x-2) - \ln(x^2+1) - \ln(2x+3)^6\right]$$

$$\ln y = \frac{1}{3}\left[\ln x + 9\ln(x+1) + \ln(x-2) - \ln(x^2+1) - 6\ln(2x+3)\right]$$

$$\frac{1}{y}\cdot y' = \frac{1}{3}\left[\frac{1}{x} + \frac{9}{x+1} + \frac{1}{x-2} - \frac{2x}{x^2+1} - \frac{6(2)}{2x+3}\right]$$

$$\therefore y' = \frac{1}{3}\left[\frac{1}{x} + \frac{9}{x+1} + \frac{1}{x-2} - \frac{2x}{x^2+1} - \frac{12}{2x+3}\right]\left[y\right]$$

$$\boxed{\therefore y' = \frac{1}{3}\left[\frac{1}{x} + \frac{9}{x+1} + \frac{1}{x-2} - \frac{2x}{x^2+1} - \frac{12}{2x+3}\right]\left[\sqrt[3]{\frac{x(x+1)^9(x-2)}{(x^2+1)(2x+3)^6}}\right]}$$

(b) **Set up an integral** to find the arc length of the graph $f(x)=2e^{3x}$ from $x=0$ to $x=e$. (8 pts.)

$$\boxed{L = \int_0^e \sqrt{1+[f'(x)]^2}\,dx}$$

8 4

$$f(x) = 2e^{3x}$$

$$f'(x) = (2)(3)e^{3x} = \boxed{6e^{3x}}$$

$2e^{3x}$

$6e^{3x}$

$$\boxed{\therefore L = \int_0^e \sqrt{1+[6e^{3x}]^2}\,dx}$$

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معتصم المعصراني

Q4. (24 points)

(a) Evaluate $\int 3^x 2^{2x}\,dx$ (6 pts.)

$$\int 3^x 2^{2x} dx = \int 3^x 4^x dx = \int 12^x dx$$

$$= \frac{1}{\ln 12} \cdot 12^x + C = \boxed{\frac{12^x}{\ln 12} + C}$$

6

$\cancel{x^2+4x+4}$

$\cancel{(x+2)^2}$

(b) Evaluate $\int \frac{dx}{\sqrt{-x^2+4x-3}}$ (10 pts.)

$$\int \frac{dx}{\sqrt{-(x^2-4x-4)-3+4}} = \int \frac{dx}{\sqrt{-(x+2)^2+1}} = \int \frac{dx}{\sqrt{1-(x+2)^2}}$$

let $u = x+2 \Rightarrow du = dx$

$$I = \int \frac{du}{\sqrt{1-u^2}} = \sin^{-1}(u) + C$$

$$= \boxed{\sin^{-1}(x+2) + C}$$

10

(c) If $f(x) = (2x-3)^3$. Show that the function f has an inverse. Find $f^{-1}(x)$. (8 pts.)

$\boxed{f(x) = (2x-3)^3}$ $f(x) = 0 \Rightarrow (2x-3)^3 = 0 \Rightarrow (2x-3) = 0$ $x = \boxed{\frac{3}{2}}$

The root

$f'(x) = 3(2x-3)^2(2) = \boxed{6(2x-3)^2}$

$f'(x) = 0 \Rightarrow 6(2x-3)^2 = 0 \Rightarrow (2x-3)^2 = 0$

$2x-3 = 0 \Rightarrow \boxed{x = \frac{3}{2}}$

3

$- - - - - \quad \frac{3}{2} \quad + + + + + +$

we notice that the function after $x = \frac{3}{2}$ is ~~increasing~~ always increasing, and if function increasing, we can conclude that the function has an inverse [and pass the horizontal line test].

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